Parallel & Distributed Graph algorithms for large graphs, practical challenges

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Who we are

I3S is the computer science laboratory of Université Côte d'Azur. It is located at the heart of Sophia Antipolis.

 COATI — theoretical and experimental aspects of graph algorithms. Software production: 3 librairies:

MascOPT network optimization (2001-) Grph computing large graphs in-memory (2010-) BigGrph platform — distributed library for computing largER graphs (2014-)

 SCALE — theoretical and experimental aspects of distributed computing. Software: ProActive, a platform for component-based computing.

COATI is hosted/supported by Inria.

Graphs, Digraphs, Hypergraphs



Figure: Weigthed graph, directed graph ad Hypergrah

- Undirected Graph : Vertices + Edges (vertices pairs) \leftrightarrow Symetric binary relation
- Directed graph : Vertices + Arcs = Couple of vertices
- Hypergraphs : (hyper)Edges = Groups of vertices

Mostly study the topology (structure) of the graph, however graphs are often *weighted* \rightarrow values labeling the vertices and arcs are also arcs and arcs are also arcs are also below the vertices and arcs are also below to be arcs are also below to be arcs and arcs are also below to be arcs are also be arcs

Graphs : becoming ubiquitous in sciences

some say pervasive ...



Graph representation, memory usage

A graph with *n* vertices *m* edges graph can be encoded (stored) as: Its *Adjacency Matrix* \rightarrow *n* \times *n* matrice. for each vertex the list of neighbors \rightarrow *n* + *m*. Structure may allow compression.





Figure: Interval graph, Union of 3 cliques.

Graph representation, memory usage (II)

Labeling of the vertices matters.

- Hypercube of dimension n + proper labeling → edges are encoded in the labels. With a random labeling edges appear as arbitrarly
- In a Tree one can always label the sons of a vertex consecutively. A node on store only the ID of its first neighbor and its degree.
- In a subgraph of a Grid one can always label the potential neighbors of a node as 0, 1, 2, 3.



Graph representation : alt representation, hidden constants

There are cases in which the natural representation is not the list of edges.

Using Alternative representation

- For a planar graph, a planar embeding (as example the list of *faces*) may be necessarly to run efficiently the algorithms.
- For an interval graph, the natural representation is to encode node as intervals.
- More generaly additional information, such as a Tree decomposition may be usefull.

Graph representation : Very large graphs

Some specifities of very large graphs

- Constants do matter, using high level abstract data structures increase the memory footprint by a large factor. Efficient solutions are often ad-hoc.
- For very large graph finding and using some hidden specific structure or compressing the graph representation may be unfeasible.
- There are many cases in which some aspect of the structure are known in advance. As example graphs in the plane or physical space, graph for which a natural partitionning do exist.

Graph Properties and Graph Algorithms

We may distinguish two different but related types of questions :

- A) Determine some properties of the graph per se.
- B) Find some properties of the graph that allow to answer to the questions of type A.

A few Graph properties, type A questions

Statistics

- degree sequence, average distance, average connectivity
- (approx) count small subgraphs, (e.g. count triangles)
- correlation and clustering $(Prob[(u, v) \in E \mid \{(u, z), (z, v)\} \in E])$

global properties

- (strongly) connected components, (approximated) Minimum Dominating Set.
- Diameter.

A few Graph properties, type (A+B) questions (II)

Approximated representation & compression

- Find a Map $f : G \to R^d$, l_1 which "preserve" the distances $\frac{1}{\rho} \leq \frac{d(x,y)}{d(f(x),f(y))} \leq \rho$ (low distorsion mapping).
- Find a simple Random Graph model such that G looks like a typical event drawn from the associated distribution (bloc models, preferential attachment models, random graph in the Euclidian plane).
- Fit G into an existing random graph model.
- Determine *clusters in G*, Find congested cuts.

Distributed Algorithm model : Bulk Synchronous Parallel (BSP)

Goal:

One wish to use a cluster of muti-core computer to implement some of these algorithms.

BSP is a message-based iterative distributed algorithm. It runs a sequence of steps. During a step:

- All messages sent at the previous steps are delivered
- All vertices in the graph are scheduled for execution

The algorithm stops when no messages remain.



Practical Implementation for large graphs ?

key performance factors

- Can we manage to fit the graph in the RAM ?
- Multi-threading ! , NEF provides CPU with 48 threads.
- 48 cores \sim PRAM with 48 processing unit.
- Can we split the data or space search without too much synchronization & communications.

Tricks : Sampling, Monte Carlo methods

- Some properties can be derived from a sample of the graph (select randomly a subset of V or a subset of E.
- \bullet \rightarrow can work on a smaler graph that fits in the RAM.
- Distant computer can work on different "chunk" of the graph.

Challenges for Distributed Algorithms

BSP framework

- Each Node is assigned a set of vertices.
- Processing phase = local computations, communication via the RAM.
- Update phase = communications, synchronization,

Performance collapses if the graph is random (or if the vertices are mapped randomly on the nodes).

Amount of communications
$$|S|\overline{d}|S| \times \frac{|V \setminus S|}{|S|} \sim \overline{d}|S|$$



Structured data \rightarrow low communication overhead

Structure is important

- There are many cases in which communications are lower.
- for grid or planar network the border of a set S is only $\Theta(\sqrt{|S|})$.
- **Bad news:** for most random networks $|\Gamma(S)| = \Theta(|S|)$.



Figure: Grid-like graph, the data is affected to 4 computing nodes.

A practical case : A snapshot of twitter

Input graph

Twitter data set (crawled by A. Legout/ INRIA -DIANA):

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- 240GB on disk, 398M vertices, 23G edges
- average degree of 58 and max degree 24,635,412

Goals

- Compute the Strongly connected Components
- Compute the number of TT_3 and $K_{2,2}$.
- Compute the diameter.

Suitability of existing frameworks

mainly Two platforms: Giraph (atop Hadoop), GraphX (atop Spark).

many flaws

- limited support for graph and programming models
- poor memory performance (GraphX cant load our large Twitter graph dataset)
- unreliable (GraphX again) steep learning curve (GraphX is written in Scala) while lacking flexibility and documentation.
- unsuitable for experimentation (slow startup, low monitoring, etc)

Our own solution : The BigGrph library

- Develloped since 2014 upon Grph (single computation flow library)
- a Java library for the manipulation of very big graphs.
- originally developed in a joint-project of Coati , Scale and Diani Inria teams: Inria provided a Research Engineer during 4 years.

Objective: running algorithms on bigdata -large graphs

BigGrph workflow

BigGrph's workflow consists of:

- 1 deploy the executable code
- 2 bootstrap the application (incremental using rsync ; takes less than a second)

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- partition the graph, by loading each piece on cluster nodes (arbitrary only)
- 4 perform the duistributed computation (BSP model) .
- 5 get the result

List of algorithms

- Single-source shortest path (Dijkstra, BFS)
- iFUB (Compute the diamter using a "few" shortest path runs).

- Page Rank
- Connected Strongly Connected components.
- Clustering coefficients, triangle counting
- Numerous stats (degrees, counting, etc)

BigGrph's performance ?

BigGrph :

- loads the graph 20x faster than Giraph
- computes BFS 3x faster than Giraph , 4x faster than GraphX
- uses 3x less memory than Giraph
- can load the big Twitter database (even on 24GB workstations) while GraphX cannot (even on 192GB cluster calculators)

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Limitation of BigGrph

Why we decided not to build upon Grph

- Abstract high level library → memory intensive.
- not designed for multi-threading.
- \blacksquare designed to hide the implementation \rightarrow not suitable for fine tuning.
- It was too complex (it took many days for our engineer to implement the SCC algorithm

Our solution

Jmaxgrph

- Just like most of others, it is written in Java, because is it the most used, taught, clean, portable, complete language/platform today
- Low memory footprint.
- Non blocking data structures
- target platform: Unix 64-bit (all Linux distributions, MacOSX,

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Use straightforward array stuctures.

Important side functionalities

The framework offer non core functionalities that are essential.

- deploy the executable code
- bootstrap the application
- partition the graph, and load each piece on cluster nodes

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- execute in parallel, communicate
- get and centralize the results

Computing strongly connected components

- Tarjan algorithm cannot be implemented transparently.
- Instead we compute local SCC \rightarrow reduce the instance size.
- The we perform 2 BFS.
- Last we call the algorithm recursively.

PerformanceComputation time on the NEF cluster:One node (512 GB RAM)7:00 hours8 nodes3:10 (gc)12 nodes2:2016 nodes2:35 (more messages)Largest SCC size = 256M vertices (64% of --V--); 141 M of size

Largest SCC size = 256M vertices (64% of -V-); 141 M of size 1; 651,000 Of size 2; ... typical random graph phenoma of isolated singleton or pairs.